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This resource book contains mathematical games, tricks, and quick activities for the classroom. Categories of activities include computation, manipulative challenges, puzzlers, picky puzzlers, patterns, measurement, money, and riddles. The computation section contains 13 classroom games and activities along with 4 manipulative challenges. Twenty-six puzzles are provided in the two puzzler sections. The book presents 10 ideas on measurement and money, 11 pattern problems, and 5 riddles. A key to each of the 69 activities, games, and tricks is included. (DDR)

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SCRATCH YOUR BRAIN WHERE IT ITCHES

BOOK C-1

MATH GAMES, TRICKS & QUICK ACTIVITIES

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DOUG BRUMBAUGH

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Scratch Your Brain Where It Itches

C-1

Math Games, Tricks, and Quick Activities

DOUG BRUMBAUGH

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CONTENTS

Computation 1

Manipulative Challenges 8

Picky Puzzlers 10

Patterns 13

Puzzlers 19

Measurement 30

Money 34

Riddles 35

Solutions 37

4

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COMPUTATION

1. Arrange the digits 1, 2, 3, 4, and 5, each digit used once per row, in a five row, five column square so that the sum of each row, column, and diagonal is 15. This would include "piecing diagonals together" so that if 3 values in a diagonal going from upper left to lower right below the major upper left to lower right diagonal were paired with the 2 value diagonal going from upper left to lower right above that same major diagonal, the sum would still be 15.

			1	
				4
3				
	2			
		5		

2. Given the four rows of numbers shown, determine which, if any, contain the square of an integer and the cube of a different integer.

A: 3 5 6 7 8

B: 4 5 6 7 8

C: 2 3 5 6 9

D: 2 5 6 7 9

E: None of above

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3. You are to take a pill every half hour. You have 3 pills to take. How long will the pills last?

4. Cover the column of addends. Uncover the list one addend at a time, starting from the top. Find the sum of all uncovered addends as the list is uncovered, writing only the grand total.

1000
40
1000
30
1000
20
1000
10

Grand Total

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5. How can

$23 + 60 = 1$?

$5 + 5 = 2$?

$6 + 24 = 1$?

6. A number is added to 1,000,000 and the result is ten times larger. Multiply the original 1,000,000 by this same number and the first amount is 10,000,000 more than the product. What is the number?

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7. Subtract 2385 from 6000 without regrouping in the standard fashion.

8. Make a true equation using 2, 3, 4, and 5.

8

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9. Statistics can be used to show there are only 30 days in a school year. What is wrong with the following calculation?

365	days in a year
-90	3 month vacation
275	
-19	school vacation days
256	
-122	days sleeping (aproximately 8 hours per day)
134	
-104	weekend days
30	days left in school

10. Divide 30 by 1/2 and add 10. What is the answer?

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11. Loop 6 digits from the group so their sum is 21.

9 9 9

5 5 5

3 3 3

1 1 1

12. Using only addition and subtraction, without changing the order of the digits, and having just three operation signs, form an equation that equals 100.

1 2 3 4 5 6 7 8 9

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13. Use six 4s to make an equation that equals 261.

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14. Six coins are arranged in an "L" shape with the horizontal part containing 3 coins and the vertical part containing 4 coins (the corner coin is counted in both the horizontal and vertical). Move one coin so there are 4 coins in both the horizontal and vertical parts of the "L."

15. In the array, "B" means there is a block in the location while "E" indicates the location is unoccupied. The task is to remove all the blocks but one. A block is removed by jumping it. All jumps are horizontal or vertical only. One block is all that may be jumped in any move. Any jump must end on an empty location. Any one block may be moved to an empty location to begin play.

B B B B

B E E B

B E E B

B E E B

12

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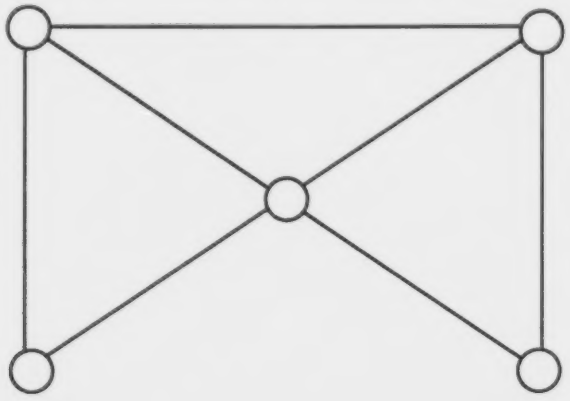
16. Five dots connected by line segments make up a game board. Two sets of two like markers are needed. One set of markers goes to each of two players. Alternating turns, each player places markers on a dot of the game board. Once all markers are placed, each player slides a marker to a vacant location with the intent of preventing the opponent from making a move. Players move only their own markers. The player who succeeds in blocking the opponent win.



17. "X" and "O" represent moveable playing pieces and "S" represents a blank space. X may move right only. O may move left only. A piece may be slid onto an adjacent blank space. A jump is over only one piece and must end in a blank space.

In the fewest possible number of moves, change

X X X S O O O

to

O O O S X X X

EXTENSION: Predict the number of necessary moves for 4 of the X pieces and 4 of the O pieces. Is there a pattern?

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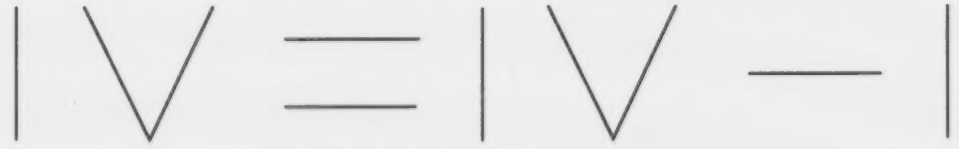
18. Make 5 + 5 + 5 = 550 a true statement by adding a single line segment.

EXTENSION: Can you create other problems like this?

19. Suppose the following equation is made up of congruent line segments. Move one segment to make this a true statement.



14

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20. You are given the following:

Add five more line segments to make 9.

21. Reposition one of the line segments to make a true equation.

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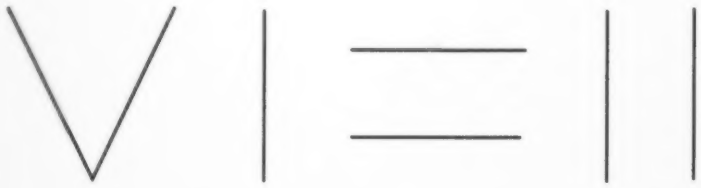
22. Move one segment to produce a true equation (making an inequality is not acceptable). The problem can be done two different ways.



23. Four congruent line segments are arranged to make a "+" sign. Move one of the segments to make a square.

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24. A logical procedure was used to generate the string of numbers below. What is the next value in the string and why? What clues could be given to lead an individual to the correct response, with the stipulation that a clue should not totally give away the solution?

1, 2, 3, 2, 1, 2, 3, 4, 2, 1, 2, 3

25. Determine the values that belong in the last two blanks. Can the pattern be continued? How far? Why or why not?

39, 13, 78, 87, 75, 25, ____, ____.

EXTENSION: What are the next two values?

39, 13, 78, 87, 75, 5, ____, ____.

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26. In the following code, each letter equals the stated value. What does I equal?

A = 2, B = 0, C = 2, D = 0, E = 3, F = 3, G = 2, H = 4.

EXTENSION: Develop some clues that would not totally give away the answer. For example, Would the answers differ if lower case letters were used? a = 2, b = 1, c = 2, etc. What would lower case "i" equal?

27. The following numbers are grouped according to some rule. What is the rule?

A = {1, 2, 6, 10, . . . }
B = {0, 4, 5, 9, . . . }
C = {3, 7, 8, 40, . . . }
D = {11, 12, 20, 30, . . . }

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28. What is the next value in the sequence: 11, 21, 1211...?

29. Assume books are known by a number only. Another worker in the library started shelving books as 8, 11, 5, 4.... You are to place book number 13 in its proper location on the shelf. Where does it go? Will 13 always go in the same place?

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30. Find a pattern for the entire set of problems where "*" is some newly defined operation:

7 * 8 gives 2

12 * 4 gives 1

2 * 1 gives 14

6 * 0 gives 11

31.

1 × 1	=	1
11 × 11	=	121
111 × 111	=	12321
1111 × 1111	=	1234321
11111 × 11111	=	?
111111 × 111111	=	?
1111111 × 1111111	=	?
11111111 × 11111111	=	?
111111111 × 111111111	=	?

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32. Where does the "Z" go and why?

A				E	F		H	I		K	L	M	N							T		V	W	X	Y
B	C	D			G			J						O	P	Q	R	S		U				

33. What are the next two values in the sequence?

1, 3, 6, 11, 19...

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34. The numbers 63, 720, 6435, 9, 252, and 9801 all belong in a given set and none of the numbers 14, 5678, 4, 47, 246, and 3541 belong in the set. Which of the numbers 36, 52, 5, 459, 3276 belong in the set and why?

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35. One year Babe Ruth had a higher batting average for each half of the season than Lou Gehrig did, but for the year, Lou Gehrig had a higher batting average than did Babe Ruth. How can that be?

EXTENSION: Can you find other examples of this problem?

36. You are given 5 beans and 4 bowls. Place an odd number of beans in each bowl.

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37. What is the fastest you have ever traveled?

38. Two children always argue when they are given a candy bar to divide between them. Even when the donor divides the candy, there are disagreements. There is a way to divide the candy to assure both children will get the same amount and there will be no arguments. How can the candy be divided?

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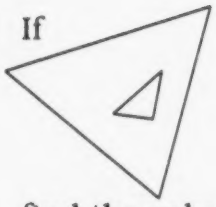
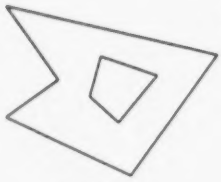
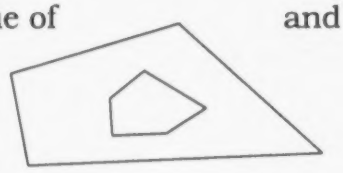
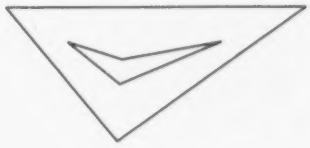
39. A Ruler had two very jealous children. They were constantly fighting over many things. Both felt that whatever they had was better than what the other had. This was especially true of the horse each owned. The Ruler decided to frustrate the children by offering a $100,000.00 prize to the owner of the horse that lost a race between the two horses with the children as riders. The race went on for days until a suggestion was made. What was the suggestion?

40. If  = 27, and if  = 1024,

find the value of  and 

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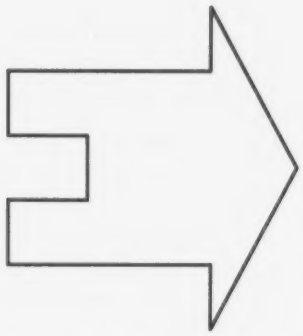
41. This shape appears on a sign you see frequently. What is the sign?



42. A half is a third of it. What is it?

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43. What can you add to 1,000,000 and always get more than if you multiplied the 1,000,000 by the same value?

44. Arrange $1000 in 10 envelopes so that any amount of money (assume counting number values only) from $1 to $1000 can be given by taking some combination of the envelopes. What amount of money is in each of the 10 envelopes?

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45. If 1.5 chickens lay 1.5 eggs in 1.5 days, how many eggs do 3.5 chickens lay in 3 days?

46. Given 10 blocks and 3 empty containers, place all 10 blocks so there is an odd number of blocks in each container. How many different ways can this problem be solved?

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47. The football goal post below shows a ball going through the uprights. Move exactly two parts of the goal post so the ball is no longer between the uprights but there is still a goal post.

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48. A person was born on July 4, 30 B.C. and died on July 4, 30 A.D. How many years did the person live?

49. If there are 12 one cent stamps in a dozen, how many two cent stamps are in a dozen?

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50. Arrange 1, 3, and 6 to form a 3-digit number that is divisible by 7.

51. Which equation is larger?

$0 + 1 + 2 + 3 + 4 + 5 =$

$5 \times 4 \times 3 \times 2 \times 1 \times 0 =$

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52. Make this a true statement by moving only one digit.

101 – 102 = 1

53. A person gives a 3-digit number in which none of the digits are repeated. That number is reversed and the smaller is subtracted from the larger. The person reveals only the ones digit of the missing addend, and you can tell the rest of the answer. How can that be?

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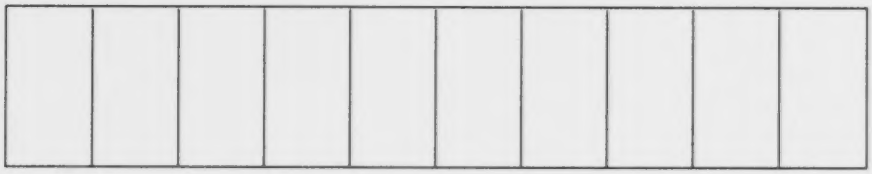
54. Given a 10 stall, single line segment horse barn, how can 9 horses be used to fill all 10 stalls?



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55. Solve the "Use a 5 gallon bucket and a 3 gallon bucket to get 4 gallons of water" problem at least 3 different ways. Assume the supply of water is not limited and only the 3 and 5 gallon containers are available for measuring and storage.

56. Two cars travel between two towns that are 200 miles apart. Car "A" averages 50 mph one way and 40 mph on the return trip. Car "B" averages 45 mph both ways. Do the two cars travel the total distance in the same amount of time?

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57. Suppose someone gives you 3 bags, 2 of which contain fool's gold weighing 1 ounce per nugget and the third containing real gold weighing 1.1 ounce per nugget. You are permitted 1 weighing to determine which is the bag containing the real gold. How can it be done?

58. A pharmacist receives a shipment of 10 bottles of the same kind of pills. Each pill is to weigh the same, and each bottle contains 1000 pills. Information is then received that each pill in one bottle is 10 mg heavier than the known weight of the pills. The pharmacist is to identify the bottle containing the heavy pills and return it and all the pills it contains to the manufacturer. What is the fewest number of weighings required to identify the bottle of heavy pills?

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59. A window is a square, one yard on a side. It admits too much light, so half of it is covered. After the covering is in place, the window is still a yard high and a yard wide, but it admits only half as much light. How can that be?

60. How long will it take to cut a 40-yard-long piece of cloth into 40 one-yard-wide pieces, allowing one minute for each cut?

61. An empty container weighs 20 kilograms. What can you put in the container to make it weigh 17 kilograms?

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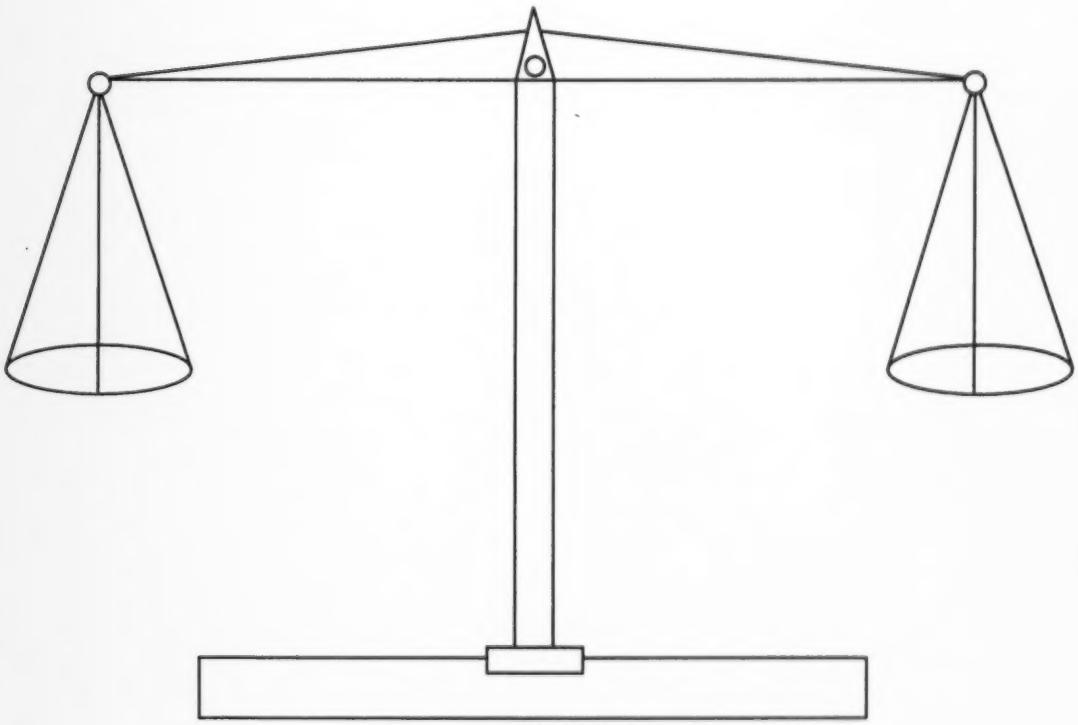
62. You are given a number of objects, all of which appear identical. In reality, all weigh the same except for one which is slightly heavier than the others. Your task is to identify the heavy object using a double pan balance upon which as many objects as desired may be placed. The desire is to be able to identify the heavy object each time the activity is done and to do it in the fewest possible number of weighings.

How many weighings does it take to identify the heavy object if given 8? How many weighings given 25? Describe the pattern that tells the number of weighings no matter the number of initial objects.



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63. You have two United States coins with a total value of $0.35 and one of them is not a quarter. What are the coins?

64. What United States coin doubles its value when its half is removed?

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65. A butcher is 6 feet 2 inches tall, wears a size 18 collar and needs size 13 shoes. What does the butcher weigh?

66. How can seven be made even?

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67. How many months have 28 days?

68. How many birthdays does the average person have?

69. What is half of 8?

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SOLUTIONS

1. 4 5 1 2 3

5 1 2 3 4

1 2 3 4 5

2 3 4 5 1

3 4 5 1 2

2. One likely choice is "B," but the directions were that the row is to contain the square of one integer and the cube of a different integer, so what are the integers squared and cubed in "B"? More than likely the response will be "2 is squared to get 4, and 2 is cubed to get 8." This violates the initial instructions of different integers being used. The next likely choice is "E," which is not correct. "B" is correct: 2 is cubed to get 8 and −2 is squared to get 4.

3. One hour. Take the first one at 4, the second at 4:30, and the last at 5.

4. It is quite common to get a response of 5000 when the total is actually 4100.

5. 23 hours + 60 minutes = 1 day, 5 fingers + 5 fingers = 2 hands, 6 days + 24 hours = 1 week

6. Zero. Add it to the end of the 1,000,000 and get 10,000,000. Multiplying gives a product of zero.

7. 6000 = 5999 + 1. Subtract 2385 from 5999 and add 1 to the missing addend to get 3615. Note that this extends to 6003 − 2385 by writing 6003 as 5999 + 4.

8. $4 + 5 = 3^2$

9. Several items are double counted; for example, some weekends occur during vacation time.

10. 30 divided by 1/2 = 30 × 2 = 60.

60 + 10 = 70

11. Turn the paper upside down.

12. 123 − 45 − 67 + 89 = 100

13. $4^4 + 4 + 4/4 = 256 + 4 + 1 = 261$

14. Stack the corner 2 coins high, using one from the vertical part.

15. Number the array

1 2 3 4

5 6 7 8

9 10 11 12

13 14 15 16

Start by moving 8 to 10. Then make the following moves, each time removing the jumped block: 9 to 11; 1 to 9; 13 to 5; 16 to 8; 4 to 12; 12 to 10; 3 to 1; 1 to 9; 9 to 11.

16. Answers will vary. One example is shown below: I have X, you have Y, and 0 is the vacant point. Your turn. You are dead since you must slide along a line segment and there is no segment between Y and O.

Scene 1

X Y

O

X Y

Scene 2

X Y

X

O Y

17. Turn the game board upside down and things are reversed. Using R for right and L for left, and thinking $X_1 = A$, $X_2 = B$, $X_3 = C$, $O_1 = D$, $O_2 = F$, $O_3 = G$. SC means Slide C. DJC means D Jump C. Right and left are not needed since X moves right only and O moves left only. SC, DJC, SE, CJE, BJD, SA, DJA, EJB, FJC, SC, BJF, AJE, SE, FJA, SA.

18. Add a line segment to the first "+" sign, making it a "4" so the problem reads 545 + 5 = 550.

19. V = IV + I

41

20. NINE

21. Note that each "+" is composed of two line segments. –I – II + III + IIII = 4.

22. XI = 11 or $\sqrt{\text{T}}$ = 1

23. Create the numeral 4.

24. The number of letters required to make the counting numbers expressed as Roman Numerals, beginning with one.

25. Divide 39 by 3. Multiply 13 by 6. Add 9 to 78. Subtract 12 from 87. Continue increasing the new value by 3, and, at the same time, the divide, multiply, add, subtract sequence. That is, initially 39 was divided by 3. Increased the 3 by 3 to get 6 and then multiplied by 13, getting 78. Increased the 6 by 3 to get 9 and added to 78, giving 87. Increased the 9 by 3 to get 12 and subtracted from 87, giving 75, etc.

EXTENSION: Divide by 3, multiply by 6, add 9, subtract 12, divide by 15, multiply by 18, etc.

26. I = 2 (or, it could be 4, depending upon the type—the number of endpoints are counted).

27. Spell out each number and count the number of letters in each word. For example, one, two, six and ten have three letters each, so these numbers are put together in group "A."

28. A) The next value is 111221 because each successive value is obtained by reading the previous value: 11 has two ones to give 2 1s or 21; 21 has one 2 and one 1 to give 1211; 1211 has one 1, one 2, and two 1s to give 111221.

B) OR, 3112 because the number of each digit is counted from lowest to highest. That is, 1211 has three 1s and one 2. The next number in this series would be 211213.

C) OR, 1231 because the number of each digit is counted from highest to lowest. That is, 1211 has one 2 and three 1s. The next in this series would be 131221.

29. The given values are in alphabetic order. Thirteen being in the same place depends on the number of books.

30. The values in each row have a sum of 17.

31. 123454321

12345654321

1234567654321

123456787654321

12345678987654321

32. Straights on top. Curves on bottom.

33. Add the preceding two values and then add 2 to that sum: 11 + 19 = 30; 30 + 2 = 32—the next number after 19; 32 + 19 = 51; 51 + 2 = 53—the next number after 32.

34. The numbers 36, 459, and 3276 belong because they are all multiples of 9.

35. RUTH
1st (H/AB = Avg): 60/200 = 0.300
2nd (H/AB = Avg) : 40/100 = 0.400
Year (H/AB = Avg):100/300 = 0.333

GEHRIG
1st (H/AB = Avg): 29/100 = 0.290
2nd (H/AB = Avg) : 78/200 = 0.390
Year (H/AB = Avg):107/300 = 0.356

36. Put bowl A inside bowl B, inside bowl C, inside bowl D. Put one bean in A. Put 2 beans in B. Put 2 beans in C. Now A contains 1 bean, B contains 3 beans, C contains 5 beans, and D contains 5 beans.

37. Add your number to 66,600 miles per hour or 107,200 kilometers per hour, which is the approximate average speed at which our earth travels in its orbit

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path through the Milky Way with the rest of our solar system. If that is not fast enough, you can add the solar system speed or the surface speed of your location on earth.

38. One child divides the candy and the other picks first.

39. Each child rides the other's horse.

40. Use the number of sides of the internal polygon as the base and the number of sides of the external polygon as the exponent: $3^3 = 27$, $4^5 = 1024$, $5^4 = 625$, and $4^3 = 64$.

41. Exit

42. 1.5

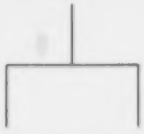
43. Any proper fraction or zero

44. $1; $2; $4; $8; $16; $32; $64; $128; $256; $489 (left over amount)

45. 1.5 eggs/1.5 chickens = 1 egg/chicken. So have (1 egg/chicken)/1.5 days = 2/3 egg/chicken/day. (2/3 egg/chicken/day) × 3.5 chickens × 3 days = 7 eggs.

46. Put 7 blocks in one container, 2 in another, and 1 in the remaining one. Then place the third container in the second. (There are 15 solutions to this problem.)

47. The new goal post is upside down. Slide the horizontal bar right so the left end is over the base of the goal post. Move left upright to the right end of the moved base and have it go down to the ground.



48. 59 years. There was no year zero.

49. 12

50. Turn the 6 upside down; 931 is divisible by 7.

51. The sum (since the product has a factor of zero.)

52. $101 - 10^2 = 1$

53. The tens digit of the missing addend will always be 9 and the sum of the ones and hundreds digit will be 9. For example, 725 reverses to 527, giving a difference of 198; 251 reverses to 152, giving a difference of 99, or 099, so the hundreds digit would be 0 if it were written.

54. Spell out N I N E H O R S E S.

55. A) Fill 3. Dump into 5. Fill 3. Top off 5. Dump 5. Pour 1 left in 3 into 5. Fill 3. Dump into 5. Now have 4.

B) Fill 5. Top off 3. Dump 3. Pour 2 left in 5 into 3. Fill 5. Top off 3. Have 4 left in 5.

C) Fill 3. Dump half (water is poured out until there is a diagonal water line in the container that goes from the bottom of the open end to the top of the closed end of the container). Now have 1.5. Fill 5. Get half of 5 as done with 3. Now have 2.5. Pour the 1.5 in with the 2.5 in the 5, giving a total of 4.

56. No. A takes 200/50 = 4 hours + 200/40 = 5 hours for a total of 9 hours. B takes 400/45 = 8.8 (8 repeats) hours or approximately 8 hours and 53 minutes for the round trip.

57. Take 1 nugget from the first bag, 2 from the second, and 3 from the third. A total weight of 6.1 means the real gold is in the first bag; 6.2 means the real gold is in the second bag; 6.3 means the real gold is in the third bag.

58. Label the bottles "A" through "J." Place one pill from "A," 2 from "B," 3 from "C,"..., 10 from "J" on a scales. The total weight should be 55 times the weight of one pill. However, since the pills in one bottle are known to be heavy, the total weight will be heavy by some amount. If bottle "D" were the heavy one, there would be 4 heavy pills in the group and

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the total weight would be 40 mg heavy. If the total weight is 70 mg heavy, it is known the heavy pills are in bottle "G."

What if the same situation exists but it is not known whether the "bad" pills are light or heavy? The pharmacist is being asked to count out and weigh 55 pills. How much time does that take? Is there a more practical way of accomplishing the task in the first place? Knowing the weight of a good pill and the weigh of the container, weigh each container individually. The bottles containing heavy pills will be heavy, so the whole task can be completed in ten weighings at most.

59. Connect the midpoints of each of the sides of the original square window, forming another square of half the original area.

60. 39 minutes. What happens if a student says to fold the cloth in half first; then cut?

61. Holes

62. TWO OBJECTS take 1 weighing; one on each side.

THREE OBJECTS take 1 weighing; one on each side. If the two balance, the one not on the balance is heavy. If the two do not balance, the heavy one is identified. Many students will want to use the third object just to be sure. This should be permitted in the hope that they will eventually determine that the moves are not needed.

FOUR OBJECTS take 2 weighings; two on each side. The heavy pair is identified. Repeat the process for two.

FIVE OBJECTS take 2 weighings; two on each side. If they balance, the remaining one is heavy. However, this would not happen each time. Thus, assume the heavy one is on the balance. One side contains the heavy one. Repeat the process for two.

SIX OBJECTS take 2 weighings. This can be done two ways. Put two objects on each side of the balance, leaving a third pair off. One of the three pairs will contain the heavy object. If the pairs on the balance do not contain the heavy object, it is in the pair not on the balance. If the heavy one is on the balance, the heavy pair is identified. Repeat the process for two with the pair containing the heavy object, or create two pairs of 3 on the balance. One of the sets will be heavy. Now, repeat the process for three.

SEVEN OBJECTS take 2 weighings; two sets of three. If the two sets balance, the remaining object is heavy. However, this would not happen each time. One set of three contains the heavy one. Repeat the process for three

EIGHT OBJECTS take 2 weighings; two sets of 3 on balance. Follow procedure for two or three, depending upon where the heavy object is. Twenty-five objects take 3 weighings; two sets of nine and one of seven. Follow the procedures for nine or seven, depending upon where the heavy object is. The pattern involves counting number powers of three: 3^1 is the last value for one weighing; 3^2 is the last value for two weighings, etc. The trick often is in determining how to do the groupings. Use the largest counting number power of three within the set that permits two or more subsets of the same cardinality.

63. The other one is a quarter and the first one is a dime

64. Half dollar. Remove the half and you have a dollar.

65. Meat

66. Delete the "s."

67. Only February has exactly 28, but all months have at least 28.

68. One

44

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69. Three if a vertical cut is made at the line of vertical symmetry; zero if a horizontal cut is made at the line of horizontal symmetry and assuming both parts are congruent; 4 if 8 is divided by 2.

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